## Enzyme Kinetics

| $7 \%$ of total |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Question | 7.1 | 7.2 | 7.3 | 7.4 | 7.5 | Total |  |
| Points | 3 | 4 | 2 | 8 | 17 | $\mathbf{3 4}$ |  |
| Score |  |  |  |  |  |  |  |

The Michaelis-Menten (MM) mechanism was introduced in 1913 to describe the kinetics of enzyme catalysis. In this mechanism, enzyme $\mathbf{E}$ catalyzes the conversion of substrate $\mathbf{S}$ to the product $\mathbf{P}$ :

$$
\mathbf{E}+\mathbf{S} \underset{k_{2}}{\stackrel{k_{1}}{\rightleftarrows}} \mathbf{E S} \xrightarrow{k_{3}} \mathbf{E}+\mathbf{P}
$$

The initial rate for an enzymatic reaction following the MM mechanism is usually given as:

$$
\begin{equation*}
v_{0}=\frac{v_{\max }[\mathbf{S}]_{0}}{[\mathbf{S}]_{0}+K_{M}} \tag{1}
\end{equation*}
$$

when the initial concentration of $\mathbf{E}$ is much lower than the initial concentration of $\mathbf{S}\left([\mathbf{E}]_{0} \ll[\mathbf{S}]_{0}\right)$. The Michaelis constant is defined as $K_{M}=\frac{k_{2}+k_{3}}{k_{1}}$. The initial rate can also be expressed as the product of the relative flow $j$ and $[\mathbf{E}]_{0}$ :

$$
\begin{equation*}
v_{0}=j[\mathbf{E}]_{0} \tag{2}
\end{equation*}
$$

Note: Questions 7.1 and 7.2 can have one, multiple, or no correct answer(s).
7.1 Choose the correct alternative form(s) of the initial rate $\left(v_{0}\right)$ expressions (1) and 3 pt (2) on the answer sheet. $[\mathbf{E S}]_{\max }$ is the maximum concentration of the $\mathbf{E S}$ complex.
7.2 Choose the pair(s) of axes ( $y$ vs. $x$ ) on the answer sheet that are expected to 4 pt give a linear plot.

Many enzymes catalyze multi- rather than single-substrate transformations. However, if the concentration of one of the substrates is much higher than that of the other substrate or it is kept constant, the MM kinetics is also valid. Here we will look at two independent enzymatic systems that follow the MM kinetics.

## Theory

## Enzymatic System I

Enzyme $\mathbf{E}$ converts substrates $\mathbf{A}$ and $\mathbf{B}$ to products $\mathbf{P}_{\mathbf{A}}$ and $\mathbf{P}_{\mathbf{B}}$. At rapid pre-equilibrium between the free enzyme and all enzyme-substrate complexes, the following $v_{0}$ equation applies:

$$
\begin{equation*}
v_{0}=\frac{k[\mathbf{E}]_{0}[\mathbf{A}]_{0}[\mathbf{B}]_{0}}{\left(K+[\mathbf{A}]_{0}\right)\left(K+[\mathbf{B}]_{0}\right)} \tag{3}
\end{equation*}
$$

$k$ is the rate constant of one of the reactions. The same equilibrium constant $K$ characterizes the dissociation of either substrate from the corresponding active site of $\mathbf{E}$.
7.3 Show that equation (3) takes the MM form (1) if the concentration of substrate 2 pt $\mathbf{B}$ is maintained at a constant value $c_{0}$. Give the expression for $v_{\text {max }}$ in this case.
7.4 Propose a kinetic scheme for the Enzymatic System I consistent with equation 8pt (3), showing all the intermediates and products. Indicate the reaction with a rate constant $k$.

## Enzymatic System II

Enzyme $\mathbf{E}$ has five active sites, each of which is specific to one of the substrates $\mathbf{S}_{\mathbf{A}}, \mathbf{S}_{\mathbf{B}}$, or $\mathbf{S}_{\mathbf{C}}$ that are selectively transformed to products $\mathbf{P}_{\mathbf{A}}, \mathbf{P}_{\mathbf{B}}$, or $\mathbf{P}_{\mathbf{C}}$, respectively. There is at least one active site for each substrate. Each active site is independent of the others.


$$
\begin{aligned}
& \mathbf{E}+\mathbf{S}_{\mathrm{A}} \underset{k_{k_{2}}}{\stackrel{k_{a 1}}{\rightleftarrows}} \mathbf{E S}_{\mathbf{A}} \stackrel{k_{a 3}}{\longrightarrow} \mathbf{E}+\mathbf{P}_{\mathbf{A}} \\
& \mathbf{E}+\mathbf{S}_{\mathbf{B}} \underset{k_{b b 2}}{\stackrel{k_{b 1}}{\rightleftarrows}} \mathbf{E S}_{\mathbf{B}} \xrightarrow{k_{b 3}} \mathbf{E}+\mathbf{P}_{\mathbf{B}} \\
& \mathbf{E}+\mathbf{S}_{\mathbf{C}} \stackrel{k_{c 1}}{\stackrel{k_{c 2}}{\rightleftarrows}} \mathbf{E S}_{\mathbf{C}} \xrightarrow{k_{c 3}} \mathbf{E}+\mathbf{P}_{\mathbf{C}}
\end{aligned}
$$

For $\mathbf{E}$, it is known that:

1. The affinity for $\mathbf{S}_{\mathbf{C}}$ is higher than for $\mathbf{S}_{\boldsymbol{B}}$.

## Theory

2. The plot of $v_{0 i}$ vs $v_{0 i} /\left[\mathbf{S}_{i}\right]_{0}$, known as Eadie-Hofstee plot, for $\mathbf{S}_{\mathbf{A}}, \mathbf{S}_{\mathbf{B}}, \mathbf{S}_{\mathbf{C}}$, with rate given per active site $\left(v_{0 i}\right)$, is shown below, but the scale and the legend are omitted.

3. When $\mathbf{E}$ is saturated with $\mathbf{S}_{\mathbf{A}}, \mathbf{S}_{\mathbf{B}}, \mathbf{S}_{\mathbf{C}}$, the catalytic turnover number (TON) for $\mathbf{S}_{\mathbf{C}}$ per site is $10200 \mathrm{~min}^{-1}$, and $2023 \mathbf{P}_{\mathbf{A}}, \mathbf{P}_{\mathbf{B}}, \mathbf{P}_{\mathbf{C}}$, product molecules in total are synthesized per second. At the same time, no more than $5.94 \cdot 10^{6}$ molecules of $\mathbf{P}_{\mathbf{A}}$ and $\mathbf{P}_{\mathbf{B}}$ are detected to form per hour.
4. At equimolar concentrations of $\mathbf{S}_{\mathbf{A}}, \mathbf{S}_{\mathbf{B}}, \mathbf{S}_{\mathbf{C}}$, which are at least 1000 times lower than corresponding $K_{M}$ values, the $\mathbf{P}_{\mathbf{A}}, \mathbf{P}_{\mathbf{B}}, \mathbf{P}_{\mathbf{C}}$, concentrations become proportional to the catalytic efficiency $\left(\varepsilon_{i}=\frac{k_{i 3}}{K_{M, i}}\right)$, and their ratio is $3: 2: 5$, respectively.
5. Two $\mathbf{E S}_{\boldsymbol{i}}$ complexes have equal rate constants for dissociation to $\mathbf{E}$ and $\mathbf{S}_{\boldsymbol{i}}$. The activation barrier for the reaction of $\mathbf{E S}_{\mathbf{C}}$ into the initial compounds is $1266 \mathrm{~J} \mathrm{~mol}^{-1}$ higher than for the reaction into the final products. Assume that the pre-exponential factors are equal for both reactions and $T=25^{\circ} \mathrm{C}$.
6. For $\mathbf{E}+\mathbf{S}_{\boldsymbol{i}}$ reaction: $k_{c 1}=1.57 \cdot 10^{7} \mathrm{M}^{-1} \mathrm{~s}^{-1}$ and $k_{a 1}=k_{b 1}$.
7.5 Fill in the table on the answer sheet and provide your calculations.

Hints:

- Use information from 1. and 2. to find the relation between $K_{M, A}, K_{M, B}$ and $K_{M, C}(<,>,=)$.
- Information from 3. and 5. allows you to complete the first column (number of active sites for each substrate) and the last row (all the constants for substrate $\boldsymbol{S}_{C}$ ) of the table. Check that the sum of active sites is equal to 5 .

